Invited talks

Jon Carlson (Athens, GA):

Thick subcategories of relative stable categories

We will discuss the problem of classifying the thick subcategories of the stable categories of modules, modulo a collection of subgroups of a finite group. This problem is important to group representation theory because it is involved with basic questions on the induction functor.

David Craven (Birmingham):

Fusion systems on groups with an abelian maximal subgroup

In this talk we will describe recent work, joint with Bob Oliver and Jason Semeraro, where we essentially classify saturated fusion systems on p-groups with an abelian subgroup of index p. We discover new exotic fusion systems, as well as unify the known examples on such groups into a general theory. As part of this classification, certain indecomposable modules for groups with a Sylow p-subgroup of order p are considered, with construction (or disproof of the existence) of such modules being heavily computer driven after an initial theoretical reduction.

Heiko Dietrich (Melbourne):

Investigating radical groups

A subgroup R of a finite group G is p-radical (p a prime) if $R = O_p(N_G(R))$, that is, if R is the largest normal p-subgroup in the normaliser $N_G(R)$. Radical subgroups play an important role in group theory and modular representation theory. In this talk we discuss some aspects of these subgroups, for example, their classification in the exceptional groups of Lie type, and the essential rank of certain classes of groups. This is joint work with Jianbei An (University of Auckland).

Bettina Eick (Braunschweig):

Cohomology and Quillen categories of finite p-groups and coclass theory

The coclass can be used as a primary invariant to investigate finite p-groups. In this talk we briefly recall this idea. Then we show how this approach can be used to investigate the Quillen categories and the Schur multiplicators of finite p-groups and to support some open conjectures on the cohomology of finite p-groups.

Graham Ellis (Galway):

Vector fields and cohomology of some crystallographic and arithmetic groups

I will discuss how Forman's discrete Morse theory can be used in computing integral cohomology rings of crystallographic groups and certain arithmetic groups (such as $\mathrm{SL}_2(\mathbb{Z}[1/m])$ and $\mathrm{SL}_4(\mathbb{Z})$).

David Green (Jena):

Ghost numbers and nilpotency degree for p-group algebras

Christensen and Wang give conjectural upper and lower bounds for the ghost number of the group algebra of a p-group. I shall report on ongoing work to verify these bounds for various families of groups. This is joint work with Fatma Aksu.

Ellen Henke (Aberdeen):

Saturated fusion systems over a Sylow p-subgroup of $\mathbf{Sp}_4(p^n)$

In this talk I will explain some general methods for classifying saturated fusion systems over a given p-group. Moreover, some sufficient conditions for a fusion system to be saturated will be stated, which can be used to construct new exotic examples. As an illustration I will report about a joint project with Sergey Shpectorov classifying saturated fusion systems over a Sylow p-subgroup of $\mathrm{Sp}_4(p^n)$ for $n \geq 1$. This leads to the discovery and construction of new exotic examples. In some small cases we use computer calculations in GAP.

Frank Himstedt (München):

On the decomposition numbers of $\mathbf{SO}_7(q)$ and $\mathbf{Sp}_6(q)$

The talk describes the completion of the ℓ-modular decomposition numbers of the unipotent characters in the principal block of the special orthogonal groups $\mathrm{SO}_7(q)$ and the symplectic groups $\mathrm{Sp}_6(q)$ for all prime powers q and all odd primes ℓ dividing $q+1$. This is joint work with Felix Noeske.

Max Horn (Gießen):

Cartan-Bott periodicity for compact forms of Kac-Moody algebras of type E_n

Kac-Moody algebras generalize finite-dimensional semisimple Lie algebras (such as $\mathfrak{sl}_n$) to the infinite dimensional, and are of great interest in theoretical physics. In particular, the Kac-Moody algebras (and groups) of type E_{10} pop up in string theory. Given an irreducible simply-laced diagram Π, we sketch the construction of a real Kac-Moody algebra $\mathfrak{g}$ of type Π, and of its 'compact form' $\mathfrak{k}$ (the fixed points of a Cartan-Chevalley involution). As an example, for $\Pi = A_n$ one has $\mathfrak{g} \cong \mathfrak{sl}_{n+1}(\mathbb{R})$ and $\mathfrak{k} \cong \mathfrak{so}_{n+1}(\mathbb{R})$. In the classic finite-dimensional case, the Lie algebras $\mathfrak{k}$ are simple, or the sum of two simple subalgebras. In contrast, in the infinite dimensional case, $\mathfrak{k}$ is far from simple, and in fact admits finite-dimensional quotients. We discuss which quotients occur in case $\Delta = E_n$ for $n \geq 3$. Interestingly, the quotients follow a nice periodic behavior, related to the classic principle of Cartan-Bott periodicity. Our proof of this boils down to a nice combinatorial argument. This is joint work with Ralf Köhl.

Gregor Kemper (München):

Computational arithmetic invariant theory

Usually invariant theory works over a field. But for some purposes it would be useful to be able to compute invariants over a ring. This talk presents new algorithms for doing that. The main tool is the Derksen ideal, named after Derksen's celebrated algorithm for computing invariants of reductive groups, and variants thereof. The talk will explain the computation of the invariant field, the geometry of Derksen ideals and cross sections, and the computation of a localization of the invariant ring. The last step is the extraction of the invariant ring from a localization, and this can be done in the arithmetic situation by using Grbner bases over a ring. The algorithm obtained in this way is applicable to the computation of invariant rings of finite groups acting on finitely generated domains over a Euclidean ring.

Viktor Levandovskyy (Aachen):

Letterplace Gröbner bases in free algebras

We present the so-called Letterplace paradigm, which is a novel view of both graded and general ideals in free associative algebras. Among other, it provides algorithms to compute two-sided Gröbner bases by performing operations in a commutative infinitely generated Letterplace ring. We will review both algorithms and their applications, in particular, to the computations of various dimensions. This talk is based on joint work with R. LaScala (Bari) and G. Studzinski (Aachen).

Nadia Mazza (Lancaster):

Endotrivial modules for some 'very important groups'

In this talk, we will start with a recap of the basics on endotrivial modules and outstanding obstacles to their classification. We will then move on to present recent techniques and results on the classification of endotrivial modules for finite groups of Lie type in non-defining characteristic and for covers of symmetric and alternating groups. These are two joint works with J. Carlson and D. Nakano, and with C. Lassueur.

Eamonn O'Brien (Auckland):

Effective algorithms for groups of Lie type

Over the past two decades much effort has been expended on developing a practical model for effective computation with matrix groups defined over finite fields. We will review this approach, with particular emphasis on simple groups, and identify how it supports the development of new algorithms.

Götz Pfeiffer (Galway):

Enumerating cyclotomic Hecke algebras

Finite Coxeter groups are particularly well behaved under enumeration of the cosets of parabolic subgroups, and the same could almost be said for finite complex reflection groups. A cyclotomic Hecke algebra is a deformation of the group algebra of a complex reflection group that plays an important role in the representation theory of finite groups of Lie Type. I will present a linear variant of the Todd-Coxeter coset enumeration algorithm for the cyclotomic Hecke algebras of certain complex reflection groups. The enumerations of such algebras over a parabolic subalgebra establish previously unknown cases of the Broué-Malle-Rouquier Freeness Conjecture, which claims that a cyclotomic Hecke algebra, like the Iwahori-Hecke algebra of a finite Coxeter group, has a basis in bijection to the elements of the group.

Peter Symonds (Manchester):

Degree bounds on homology and a conjecture of Derksen

We give a counterexample to a conjecture of Derksen concerning syzygies of rings of invariants. We also prove a modified version of the conjecture and some general results giving bounds on syzygies. This is joint work with M. Chardin.

Contributed talks

Antonio Diaz-Ramos (Malaga):

A new spectral sequence for fusion systems

We introduce a new spectral sequence to compute the cohomology ring of a fusion system with a strongly closed subgroup. This generalizes the Lyndon-Hochschild-Serre spectral sequence and applies to finite simple groups and exotic fusion systems with a strongly closed subgroup. As an example computation, we describe the cohomology ring of the second Janko group J_2 with coefficients in the field of 3 elements. It turns out that this ring is Cohen-Macaulay and its quotient by the polynomial part satisfies Poincaré duality and is the cohomology of a manifold. Another potential computation are the cohomology rings of a family of exotic fusion systems over a family of Sylow 3-subgroups of maximal nilpotency class. The smallest of these 3-subgroups is the Heisenberg group and we have recently proven that all of them have isomorphic cohomology rings (as predicted by Carlson's conjecture on the cohomology of p-groups of fixed coclass).

Deniz Erdemirci-Erkus (Izmir):

On generalized invariants and p-groups

Let V be a faithful representation of a finite group G over a field of characteristic p. An element f in $\mathbb{F}[V]$ is called a generalized invariant, if for each non-identity $\sigma \in G$, there exists a positive integer ℓ such that $(\sigma - 1)^\ell(f) = 0$, provided that $(\sigma - 1)^\ell$ is not zero in the group algebra $\mathbb{F}G$. In this talk, we will explain the structure of generalized invariants and its relation with the group structure. This is joint work with Uğur Madran.

Neil Saunders (Bristol):

Exceptional permutation groups of order p^5

The minimal permutation degree of a finite group G is the smallest non-negative integer n such that G embeds inside $\mathrm{Sym}(n)$. This invariant is easy to define but very difficult to calculate in general. Moreover, it doesn't behave well under algebraic constructions such as (semi)direct product and homomorphic image. For example, it is possible for the minimal degree of a homomorphic image to be strictly larger than that of the group — such groups are called 'exceptional'. In this talk, I will describe how this invariant maybe calculated by a greedy algorithm for nilpotent groups and report on recent work with Britnell and Skyner on the classification of exceptional groups of order p^5.

Jason Semeraro (Bristol):

A generalization of the Z^*-theorem

Glauberman's Z^*-theorem and analogous statements for odd primes show that, for any prime p and any finite group G with Sylow p-subgroup S, the centre of G is determined by the fusion system $\mathcal{F}_S(G)$. Building on these results, we show a statement that can be considered as a generalization: For any normal subgroup N of G, the centralizer $C_S(N)$ is expressed in terms of the fusion system $\mathcal{F}_S(G)$ and its normal subsystem induced by N. This is joint work with Ellen Henke.

Peter Webb (Minneapolis):

Software and algorithms for representations of algebras and categories

Software for representing categories is described, as well as an algorithm for computing almost split sequences for representations of algebras.